Exercises – Algorithms for Genome Rearrangement

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http://wiki.techfak.uni-bielefeld.de/gi/Teaching/2015summer/gr

Exercise List 4 — 04.05.2014

Discussion of exercises on: 11.05.2014

Exercise 1 **(2 Points)**

Given the signed permutation

$$\pi = (\ +3\ \ +5\ \ +4\ \ +6\ \ +2\ \ +1\ \ +7\ \ -8\ \ +9)$$

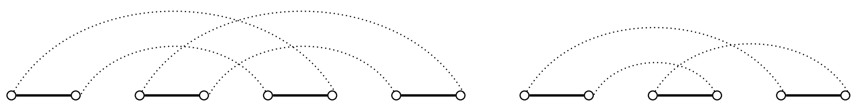
(a) How many components does the graph $BP(\pi)$ has, and of which type?

(b) What is the reversal distance?

(c) Find a sequence of reversals that transform all unoriented components into oriented components.

Exercise 2 **(2 Points)**

Consider the following breakpoint graph.

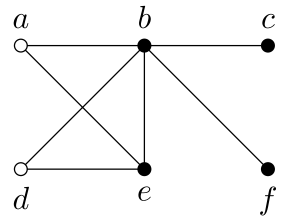


(a) What is the reversal distance for this breakpoint graph?

(b) Find a permutation π that has this breakpoint graph.

Exercise 3 **(2 Points)**

Suppose that a given permutation has the following overlap graph:



(a) What is the vertex with maximum score? Apply the reversal defined by this vertex, update the overlap graph, and repeat the process until the permutation is sorted.

(b) Can you find a breakpoint graph that corresponds to the overlap graph in the figure?

Exercise 4 **(2 Points)**

The number of possible (unsigned) permutations over $\{1, 2, \ldots, n\}$ is $n!$. Obviously, there exist bijective mappings between the numbers $1, 2, \ldots, n!$ and permutations over $\{1, 2, \ldots, n\}$. Find such a mapping that is computable in both directions in polynomial time. *Tip: Google is your friend.*

Factoradic numbers:
$(i_k i_{k-1} \dots i_2 i_1 i_0)$ represents the integer $i_k k! + i_{k-1}(k-1)! + \cdots + i_2 2! + i_1 1! + i_0 0!$.
Now, a permutation π is represented by an array A of size n where position $A[i]$ the number of indices $i' > i$ with values $\pi[i']$ smaller than $\pi[i]$:

1 2 3 4 – 0 0 0 0
1 2 4 3 – 0 0 1 0
1 3 2 4 – 0 1 0 0
1 3 4 2 – 0 1 1 0
1 4 2 3 – 0 2 0 0
1 4 3 2 – 0 2 1 0
2 1 3 4 – 1 0 0 0
2 1 4 3 – 1 0 1 0
2 3 1 4 – 1 1 0 0
2 3 4 1 – 1 1 1 0
2 4 1 3 – 1 2 0 0
2 4 3 1 – 1 2 1 0
3 1 2 4 – 2 0 0 0
...

The entries in the ith last column range from 0 to $i-1$. All rows are different.
Interpreted as factoradic numbers, we get a unique mapping.
Quadratic-time calculations are simple. Linear-time calculations are more difficult, one direction is shown in the following, the other direction is more complicated.
See `http://en.wikipedia.org/wiki/Permutation` (Numbering permutations):

```
function permutation(k, s) {
  var int factorial:= 1;
  for j = 2 to length(s) {
    factorial := factorial* (j-1);
    swap( s[j - ((k / factorial) mod j)], s[j]);
  }
  return s;
}
```

Exercise 5 **(3 Points)**

After a *cycle merge* reversal, that is, a reversal defined by black edges in two different cycles, the two cycles are merged into one. Prove that this new cycle is always oriented.