Fun with algorithms

First Session - Winter 2015-2016

Liren Huang
Damianos Melidis

Faculty of Technology, Bielefeld University

October 22, 2015

Introduction - 1

Fun with algorithms $\Rightarrow$

Objectives

1. Algorithms $\rightarrow$ Practise scientific reading, presenting and writing about an interesting algorithm
2. Fun $\rightarrow$ Enjoyment!
3. Credit points
4. Others

Introduction - 2

Algorithms

1. Each selects one publication (Starting week)
2. prepares in 2 weeks - rehearsal in the 2nd week
3. presents in the 3rd week
4. hands in the final report ($\sim$ 5 pages) within 3 weeks after presentation

Introduction - 3

Fun!

2a Enjoy reading an algorithm solving classic Nintendo® games

2b Idea to play a game related to the algorithm

Credits

3a You gain 4 credit points

3b You gain 1 credit point by implementing an algorithm of your choice!

Others

4a Be active during everyone else's presentation

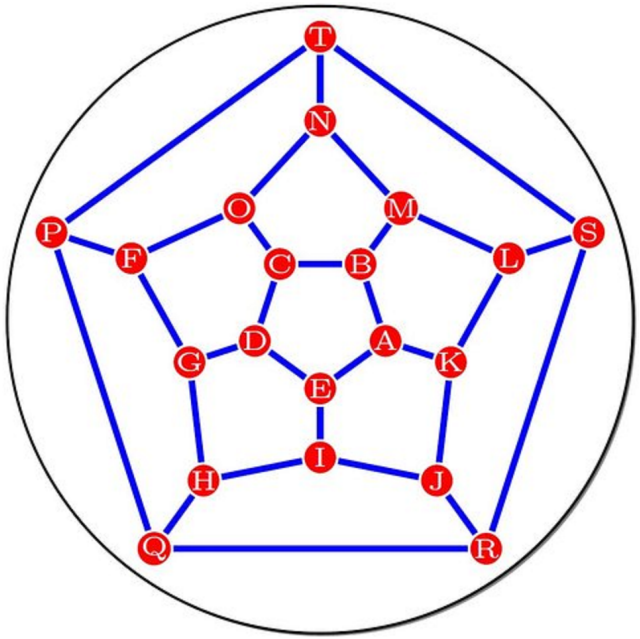
4b Ask us for support

Proposed Schedule

Week (Thursday)	1st hour	2nd hour
1 (22.10)	Preliminaries	Paper selection
2	How to read	How to write & present
3	1st rehearsal	-
4	1st presents	2nd rehearsal
5	2nd presents	3rd rehearsal
6	3rd presents	4th rehearsal
7	4th presents	5th ... 1st hands in report
8	5th presents	6th .. 2nd hands in report
9 (17.12)	6th presents	3rd hands in report

Table 1: Schedule

Icosian game - Introduction



Source: http://graphics8.nytimes.com/images/2014/10/02/crosswords/icosian/icosian-blog480-v2.jpg

The Icosian Game
by Sir. William Rowan Hamilton

Cities Names:

B Brussels	**N** Naples
C Canton	**P** Paris
D Delhi	**Q** Quebec
F Frankfort	**R** Rome
G Geneva	**S** Stockholm
H Hanover	**T** Toholsk
J Jeddo	**V** Vienna
K Kashmere	**W** Washington
L London	**X** Xenia
M Moscow	**Z** Zanzibar

Source: free pdf from http://www.springer.com/978-1-4939-0304-7

Fun with algorithms! WInter 2015, Uni-Bielefeld

Figure 1: The icosian game [1].

In 1857 Sir William Rowan Hamilton introduced this game:

1. Find a Hamiltonian cycle passing 20 cities
2. Find a Hamiltonian path starting from one city finishing to another (for example Rome $\rightarrow$ Jeddo)

[1] The New York Times' article

Icosian game - Plan of attack

Both problems are NP-complete
Need to show:

1. Hamiltonian path $\Leftrightarrow$ Hamiltonian cycle
2. Hamiltonian cycle $\in$ NP-complete
   - 2a Polynomial verifier for a given solution
   - 2b Hamiltonian cycle reduces to travelling salesman problem

*Adapted from Hamiltonian path Wiki

Icosian game - Equivalence of problems

Let G(V,E) be constructed using the vertices and edges of the icosian game

1. Hamiltonian path $\Leftrightarrow$ Hamiltonian cycle
   - 1a Hamiltonian path $\Rightarrow$ Hamiltonian cycle
     Consider a graph G_p (V_p,E_p) where a Hamiltonian path exists, then create new G_c ($V_p \cup$ v, E_c) adding a new vertex v such that v is connected with all u $\in$ V
   - 1b Hamiltonian cycle $\Rightarrow$ Hamiltonian path
     Consider a graph G_c (V_c,E_c) where a Hamiltonian cycle exists, then create new G_p (V_c - v, E_p), removing the vertex v which is the start and end of the cycle

$\Longrightarrow$ From now, consider only the complexity of Hamiltonian cycle

Icosian game - Complexity of Hamiltonian cycle

Let G(V,E) be constructed using the vertices and edges of the icosian game

2a A verifier given a hamiltonian cycle c in G can start from an abritrary vertex v, traverse each vertex of the path, in linear time of vertices of the path.
If the vertex v is visited only twice and each other vertex in V is visited only once, then it accepts this solution, otherwise rejects it

2b 3-SAT[2] $\leq_P$ Hamiltonian cycle

[2] SAT wiki

Icosian game - Proof of existence

Theorem (Smith)[3]

If G is a d-regular graph where d is odd and $e \in E(G)$, then there are an even number of Hamiltonian cycles in G which pass through the edge e

[3] Lectures notes in graph theory by Matt DeVos

Icosian game - Further reading

1. Visualization of Hamiltonian cycle in icosian game
2. google search - Hamiltonian Walks Springer (free download)
3. Hamiltonian cycle reduction to 3-SAT

Contacts

Liren Huang

* huanglr [at] cebitec [.] uni-bielefeld [.] de
* U10-120

Damianos Melidis

* dmelidis [at] cebitec [.] uni-bielefeld [.] de
* U10-144