Exercises – Phylogenetics

Exercise List 11 — 19.01.2016

Due to: 26.01.2016

Exercise 1 Revision: Basics on Probability. (5 Points)

Let $\Omega = \{1, 2, \ldots, 6\}$ be the sample space of a die with the uniform distribution Pr and $(\Omega_2, \text{Pr}_2) = (\Omega, \text{Pr}) \times (\Omega, \text{Pr})$ be the sample space of two independently thrown dices. We can distinguish between the two dices (let's call them dice 1 and dice 2) and we write the elementary events as (e_1, e_2).
Solve the probabilities for the following events in Ω_2:

(a) Dice 2 shows a 4.

(b) The sum of both dices is 8.

(c) Dice 1 shows an odd number.

(d) Dice 1 and dice 2 form a double.

(e) Dice 1 shows an even number and dice 2 shows a number between 4 and 6.

Exercise 2 Modeling of sequence volution. (7 Points)

Consider the following model of nucleotide substitution:

$$\pi = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$$

$$Q = \begin{pmatrix} -5/8 & 3/8 & 1/8 & 1/8 \\ 3/8 & -5/8 & 1/8 & 1/8 \\ 1/8 & 1/8 & -5/8 & 3/8 \\ 1/8 & 1/8 & 3/8 & -5/8 \end{pmatrix}$$

$$P(t) = \begin{pmatrix} 1 - (x + 2y) & x & y & y \\ x & 1 - (x + 2y) & y & y \\ y & y & 1 - (x + 2y) & x \\ y & y & x & 1 - (x + 2y) \end{pmatrix}$$

where

$$x = \frac{1 - 2\exp(-t) + \exp(-t/2)}{4}, \; y = \frac{1 - \exp(-t/2)}{4}$$

(a) At how much PEM is the model calibrated?

(b) Recalibrate the corresponding matrix to 1 PEM.
    (You're **not** supposed to calibrate both matrices to 1 PEM.)

(c) At how much PAM is the model calibrated?

(d) Recalibrate the corresponding matrix to 1 PAM.
    (You're **not** supposed to calibrate both matrices to 1 PAM.)