Exercises – Phylogenetics

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https://gi.cebitec.uni-bielefeld.de/Teaching/2018summer/Phylogenetik

Exercise Sheet 1 — 12.04.2018

Due: 19.04.2018

Task 1 Properties of trees. (2 Points)

To prove the equivalence of n statements $S_1 \Leftrightarrow S_2 \Leftrightarrow \cdots \Leftrightarrow S_n$, it is sufficient to show: $S_1 \Rightarrow S_2 \Rightarrow \cdots \Rightarrow S_n \Rightarrow S_1$. By this circular argumentation, we do not need to show *all* pairwise equivalence relations.

Let $G = (V, E)$ be a undirected graph. Apply the above technique to prove that following statements are equivalent.

(a) G is a tree.
(Use the definition of a tree given by the lecture notes.)

(b) Every pair of nodes $\{v_1, v_2\} \in \binom{V}{2}$ is connected by a unique, simple path.

(c) G is minimal connected, i.e., for all $e \in E$: if e is removed, the resulting graph $G' = (V, E \setminus \{e\})$ is not connected.

(d) G is connected and $|E| = |V| - 1$.

(e) G is free of cycles and $|E| = |V| - 1$.

(f) G is maximal free of cycles, i.e., for all $e \in \left(\binom{V}{2} \setminus E\right)$: if e is added to E, the resulting graph $G' = (V, E \cup \{e\})$ contains a cycle.

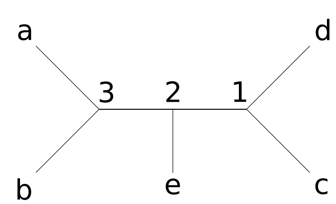
To make that task a little bit easier, we split the steps of the proof according to your students registration number (Matrikelnummer). Take the **last digit** of your registration number and do the two parts of the proof.

Digit	Parts of proof	
0–2	(a)⇒(b),	(b)⇒(c)
3–5	(c)⇒(d),	(d)⇒(e)
6–9	(e)⇒(f),	(f)⇒(a)

Hint: Some parts are proper for a direct proof, other parts for a proof by contradiction, and sometimes it makes sense to show $\neg(j) \Rightarrow \neg(i)$ instead of $(i) \Rightarrow (j)$.

Task 2 Rooted and unrooted trees. (2 Points)

(a) Consider the following tree. Add a new root at the edge `{a,3}`, draw the resulting tree, and give the corresponding NEWICK notation.



(b) Draw the tree to the following NEWICK notation:

(E,((B,C)1,(A,D)2)3)4;