

Advanced Sequence Analysis  
Summer 2019

Exercises

Number 11 (28.06.2019), Discussion: 05.07.2019

1. Find all maximal repetitions in the word 10101001010010100101001011110.
2. Prove the following theorem (Fine and Wilf): If a word  $w$  has periods  $p_1$ ,  $p_2$ , and  $|w| \geq p_1 + p_2 - \gcd(p_1, p_2)$ , then  $\gcd(p_1, p_2)$  is also a period of  $w$ .
3. Prove the “Lemma of Three Squares”:  
Let  $x^2$ ,  $y^2$ ,  $z^2$  be three primitively rooted squares, such that  $|x| > |y| > |z|$ ,  $z^2$  is a prefix of  $y^2$  and  $y^2$  is a prefix of  $x^2$ . Then  $|z| + |y| \leq |x|$ .  
(*Hint:* It helps knowing that  $|y| - |z|$  is a period of  $z$  and  $|x| - |y|$  is a period of  $y$ .)
4. Enumerate all primitively rooted squares of the Fibonacci word  $f_6$ .