

# Exercises – Phylogenetics

Universität Bielefeld, SS 2019

Dr. Roland Wittler, M. Sc. Tizian Schulz

<https://gi.cebitec.uni-bielefeld.de/Teaching/2019summer/Phylogenetik>

## Exercise Sheet 1 — 04.04.2019

Due: 11.04.2019

### Task 1

(0 Points)

Write down your name on each solution. If you have to use multiple sheets, staple them together. This “exercise” applies to every exercise list.

### Task 2 Properties of trees.

(2 Points)

To prove the equivalence of  $n$  statements  $S_1 \Leftrightarrow S_2 \Leftrightarrow \dots \Leftrightarrow S_n$ , it is sufficient to show:  $S_1 \Rightarrow S_2 \Rightarrow \dots \Rightarrow S_n \Rightarrow S_1$ . By this circular argumentation, we do not need to show *all* pairwise equivalence relations.

Let  $G = (V, E)$  be a undirected graph. Apply the above technique to prove that following statements are equivalent.

- (a)  $G$  is a tree.  
(Use the definition of a tree given by the lecture notes.)
- (b) Every pair of nodes  $\{v_1, v_2\} \in \binom{V}{2}$  is connected by a unique, simple path.
- (c)  $G$  is minimally connected, i.e., for all  $e \in E$ : if  $e$  is removed, the resulting graph  $G' = (V, E \setminus \{e\})$  is not connected.
- (d)  $G$  is connected and  $|E| = |V| - 1$ .
- (e)  $G$  is acyclic and  $|E| = |V| - 1$ .
- (f)  $G$  is maximally acyclic, i.e., for all  $e \in \left(\binom{V}{2} \setminus E\right)$ : if  $e$  is added to  $E$ , the resulting graph  $G' = (V, E \cup \{e\})$  contains a cycle.

To make that task a little bit easier, we split the steps of the proof according to your students registration number (Matrikelnummer). Take the **last digit** of your registration number and do the two parts of the proof.

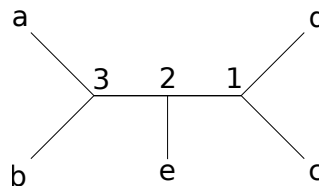
Digit	Parts of proof
0	(a) $\Rightarrow$ (b), (e) $\Rightarrow$ (f)
1	(b) $\Rightarrow$ (c), (f) $\Rightarrow$ (a)
2	(c) $\Rightarrow$ (d), (a) $\Rightarrow$ (b)
3	(d) $\Rightarrow$ (e), (b) $\Rightarrow$ (c)
4	(e) $\Rightarrow$ (f), (c) $\Rightarrow$ (d)
5	(f) $\Rightarrow$ (a), (d) $\Rightarrow$ (e)
6	(a) $\Rightarrow$ (b), (e) $\Rightarrow$ (f)
7	(b) $\Rightarrow$ (c), (f) $\Rightarrow$ (a)
8	(c) $\Rightarrow$ (d), (a) $\Rightarrow$ (b)
9	(d) $\Rightarrow$ (e), (b) $\Rightarrow$ (c)

Hint: Some parts are proper for a direct proof, other parts for a proof by contradiction, and sometimes it makes sense to show  $\neg(j) \Rightarrow \neg(i)$  instead of  $(i) \Rightarrow (j)$ .

### Task 3 Rooted and unrooted trees.

(2 Points)

- (a) Consider the following tree. Add a new root at the edge {b,3}, draw the resulting tree, and give the corresponding NEWICK notation.



- (b) Draw the tree to the following NEWICK notation:

(A,(B,((C,D)4,E)3)2)1;