DRAWING THE BOUNDARY BETWEEN SUBJECT AND OBJECT: COMMENTS ON THE MIND-BRAIN PROBLEM

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ABSTRACT. Physics says that it cannot deal with the mind-brain problem, because it does not deal in subjectivities, and mind is subjective. However, biologists (among others) still claim to seek a material basis for subjective mental processes, which would thereby render them objective. Something is clearly wrong here. I claim that what is wrong is the adoption of too narrow a view of what constitutes 'objectivity', especially in identifying it with what a 'machine' can do. I approach the problem in the light of two cognate circumstances: (a) the 'measurement problem' in quantum physics, and (b) the objectivity of standard mathematics, even though most of it is beyond the reach of 'machines'. I argue that the only resolution to such problems is in the recognition that closed loops of causation are 'objective'; i.e. legitimate objects of scientific scrutiny. These are explicitly forbidden in any machine or mechanism. A material system which contains such loops is called 'complex'. Such complex systems thus must possess non-simulable models; i.e. models which contain impredicativities or 'self-references' which cannot be removed, or faithfully mapped into a single coherent syntactic time-frame. I consider a few of the consequences of the above, in the context of thus redrawing the boundary between subject and object.

Key words: causality, complexity, computability, measurement, mind-brain problem, objectivity, subjectivity

INTRODUCTION

The discussion to follow will focus on the obvious irreconcilability of the two statements below. The first of them is taken from a rhapsodic paean to reductionism; it was penned over two decades ago, but could have been written yesterday:

> ...life can be understood in terms of the laws that govern and the phenomena that characterize the inanimate, physical universe, and indeed, at its essence, life can be understood *only* in the language of chemistry ... only two truly major questions remain shrouded in a cloak of *not quite* fathomable mystery: (1) the origin of life ..., and (2) the mind-body problem; i.e. the physical basis for self-awareness and personality ([1]; emphasis added).

The second was written by a physicist, and expresses his view of what physics is about:

> [Max] Planck designated in an excellent way ... the goal of physics as the complete separation of the world from the individuality of the structuring mind; i.e. the emancipation of anthropomorphic elements. That means: it is the task of physics to build a world which is foreign to consciousness and in which consciousness is obliterated [2].

Here we see, in a nutshell, the roots of the real problem. Handler [1] claims to be seeking the physical, material basis of things (life and mind) which physics has extruded. According to Bergman [2], neither life nor mind can be either subject or object in a true science of material nature; of *objective* reality. At best, according to this view of *physics*, life and mind are only epiphenomena; *facons de parler*; mere names given to certain classes of objective, material events and behaviors; devoid of any causal agency.

As I shall argue below, the real issue here is not a technical, but a conceptual matter. The central concept at issue is *objectivity*, for this alone determines whether something falls into the realm of science or not. As it stands now, we must say that organisms are objective but life is not; brains are objective but mind is not. Accordingly there can be a science of organisms, and a science of brains, but no science of life or of mind. Conversely, if life and mind are to be made objects of *scientific* study, it is our conceptions of objectivity which must change.

The identification of 'objectivity' with what is independent of, or invariant to (these are not the same) perceivers, or cognizers, or observers, is what has led to the current infatuation with the *machine* as simulacrum of both life and mind. Roughly speaking, if a machine can 'do' something, that is *prima facie* evidence of its 'objectivity', and of its admissibility into science. Hence the conflation of mechanism with what is 'objective', and the relegation of anything non-mechanistic to the realm of the subjective, the *ad hoc*, the vitalistic, the anthropomorphic; in short, beyond the pale of science.

I shall argue, to the contrary, that mechanism in any sense is an inadequate criterion for 'objectivity'; that something can be objective (and hence a candidate for scientific scrutiny) without being in any sense a mechanism. That is, the perceived dichotomy between 'mechanism' and 'vitalism' (i.e. denial of the former means affirmation of the latter) is is a false one.

Under these circumstances, we shall argue (a) that Handler's "not quite fathomable" problems are so because of a mistaken equation of mechanism with objectivity; accordingly, it will also follow that the physics Bergman describes, in which "consciousness is obliterated", is only a small part of a full science of material nature or objective reality.

OBJECTIVITY

The world "foreign to consciousness", which Bergman evokes in the above citation, comprises the external world of events and phenomena; the *public* world. The world of mind and consciousness, on the other hand, is a private world; what happens in it, and in whatever it touches, constitutes the *subjective*. We should notice that such a partition of our immediate universe of impressions and percepts, into a public part and a private part, is itself private. There is no public test for 'publicity'; it is something privately posited.

The entire concern of science, especially theoretical science, has been to make public phenomena apprehensible to the private, cognizing mind. The 'mind-brain problem' represents an attempt to go the other way; to pull the private world into the public one, and thus to make the mind apprehensible to itself by expressing it in phenomenal terms.

In one way or another, the concept of *causality* dominates our conception of what transpires in the external, public world. In broadest terms, causality comprehends a system of entailments, which relate the events and phenomena occurring therein. The concept itself is due to Aristotle, who associated it with the answers given to the question "Why?"; indeed, to Aristotle, science itself was the systematic study of "the why of things", and hence entirely concerned with elucidating such causal relations.

Intuitively, we think of something as 'objective' if its perception, or cognition, plays no causal role in its *entailment*; i.e. answers no question "Why?" about it. Stated another way: the private perceptions or cognitions about the thing have no public counterparts or manifestations in the thing itself. It is only in this sense that 'objective' things are foreign to consciousness.

In practice, 'objectivity' has become more narrowly construed still, especially in biology. It has come to mean, not only independence from a perceiving mind, but independence from any larger system whatever, public or private, of which it may (or may not) be a part; hence, independence from any *environment*. Accordingly, the notion of function, which depends on just such a larger system, is disallowed any objective status; likewise any notion of emergence. This is one of the wellsprings of Reductionism; that it is only 'objective' to explain wholes in terms of parts; never parts in terms of wholes.

So, for example, a molecule of DNA, or even just its *sequence*, are accorded objective status; they can be removed from their original environment without affecting these characteristics, and hence we can ask "Why?" about them without ever invoking the organism from which they came. The 'Mendelian gene', on the other hand, had no such status, since it had to be defined exclusively in *functional* terms, through manifestations in phenotype. On similar grounds, the Behaviorist school in psychology has stoutly denied 'objectivity' to what it calls 'internal mental states', reserving that term entirely for functional input-output (stimulus-response) characteristics analogous to those of classical genetics.

Perhaps the strongest statement about 'objectivity' was given by Jacques Monod:

> The cornerstone of the scientific method is the postulate that nature is objective. In other words, the systematic denial that 'true' knowledge can be gotten at by interpreting phenomena in terms of final causes ... the postulate of objectivity is consubstantial with science ... there is no way to be rid of it ... without departing from the domain of science itself ([3], p. 21).

We shall come to what "final causes" means shortly.

The upshot of all of this is the following. First, we must allow public events to have only public explanations, public causes; never private ones. Second, in that public arena, causal chains must always flow from parts to wholes; never from wholes to parts. This, and only this, constitutes the domain of 'objective reality'.

In Aristotelian terms, then, 'objectivity' means in practice an exclusive concern with what can be accommodated entirely within *only three* of his four posited causal categories. Notions of material, formal, and efficient causation alone are necessary and sufficient for the external, public world; final causation is excluded, and relegated at best to the private, subjective one. Accordingly, the world of the 'objective' allows its events to be displayed along one single, coherent time-frame, ('real' time) in which causal entailment flows from past through present to future, and never the other way. Thus arises what I have called the Zeroth Commandment: thou shalt not allow future state to affect present change of state. Systems in which this happens are called *anticipatory* (cf. [4]) and accordingly are dismissed as 'acausal'.

The main consequence of these views of 'objectivity' is that *closed causal loops are forbidden.* Only chains or trees are admitted into the 'objective' world, arrayed like branches along the trunk of that single, coherent all-encompassing time-frame.

Our argument will be that, if closed loops of causation are denied objective status *per se*, then the 'mind-brain problem' falls irretrievably outside of science. Systems without closed causal loops are, broadly, what I have called *machines* or *mechanisms*, or more generally, *simple systems* (cf. [5]). What makes simple systems simple in this sense is, roughly, that they are so weak in entailment that there is not enough to close such a causal loop. As we shall see, the alternative is not a 'subjective' world, immune to science, but a world of *complex* systems; i.e. of systems which are not simple.

CAUSALITY AND 'THE MEASUREMENT PROBLEM'

As we have used it above, the term "causality" pertains to entailment relations which can be established between public events. Especially between events occurring 'here and now', and other events occurring 'there and later (or earlier)'. In physics, this causality has always been ineluctably connected with the notion of *state* (or, more precisely, with the state of a 'system' at a particular instant). Intuitively, the information comprising such a state consists of all you need to know to answer every question "Why?" about the system's behavior. It also connotes a notion of parsimony; that a state is the least you need to know to answer these questions. The determination of state, however, requires *observation*; it requires measurement.

In these terms, the partition between what is objective and what is not transmutes into the partition between a 'system' and an *observer*. That partition has been most extensively and most urgently discussed in the context of quantum theory, and that from its very earliest days. That continuing discussion has been, to say the least, inconclusive. But several things have emerged from it, which bear on our present considerations: (1) *where* the partition between 'system' and observer is drawn is entirely arbitrary; (2) *wherever* the partition is drawn, it always seems to leave some 'physics' (i.e. something public or objective) on the wrong side of it; (3) the more we admit into the objective or public side, the more porous the partition itself seems to become.

Concern with 'the Measurement Problem' was, for instance, a central theme of von Neumann's [6] early monograph on quantum theory, so much so that at least a third of its pages are devoted to it. The root of the problem, of course, lay in the fact that, at the quantum level, observation apparently had to be invoked to account for the result of observation. This immediately flies in the face of objectivity itself, where as we have seen, something is to be counted as objective only if it is (causally) independent of observation; if observing something answers no question about why it is what it is. At the very least, measurement appears to invoke a causal flow from a whole (system + observer) to a part (system alone) which, as we noted earlier, is stoutly denied 'objective' status.

Let us briefly consider von Neumann's treatment of these matters, because they are directly pertinent, and because they have provided the basis for many subsequent discussions. First, as to the necessity to make *some* partition between observer and the observed; between the objective and the subjective:

> It is inherently entirely correct that the measurement or the related process of the subjective perception is a new entity relative to the physical environment and is not reducible to the latter. Indeed, subjective perception leads us into the intellectual inner life of the individual, which is extra-observational *by its very nature* ... ([6], p. 418; emphasis added).

Then he goes on to say

> ... we *must* always divide the world into two parts, the one being the observed system, the other the observer. The boundary between the two is arbitrary to a very large extent ... this boundary can be pushed arbitrarily deeply into the interior of the body of the actual observer ... but this does not change the fact that in each method of description the boundary *must* be put somewhere, if the method is not to proceed vacuously ... ([6], p. 420).

Von Neumann, following Bohr in this, invokes the necessity for such a partition to justify a dualism mandated by quantum theory itself to deal with the 'measurement process'. Namely, a quantum-theoretic system will behave causally as long as it is not being observed, but 'non-causally' (i.e. statistically) otherwise. The 'causal' aspect is embodied in, say, the system's own Schrödinger equation governing autonomous change of state. But this equation no longer holds when change of state arises from non-autonomous interactions, e.g. with measuring instruments. These latter create dispersions, irreversibilities, and dissipations, which have no classical counterparts, and which make the 'state' of the system a much more problematic concept, in terms of its causal content (i.e. the questions "Why?" it can answer) than it was previously.

Actually, von Neumann's primary concern is to affirm quantum theory by denying causality; relegating it to the status of a simple macroscopic illusion:

> ... the position of causality in modern physics can therefore be characterized as follows: In the macroscopic case there is no *experiment* which supports it, and none can be devised because the apparent causal order of the world in the large ... has *certainly* no other cause than the 'law of large numbers' and it is completely independent of whether the natural laws governing the elementary processes are causal or not ... The question of causality could be put to a true test *only* in the atom, in the elementary processes themselves, and here everything in the present state of our knowledge militates against it ... We may [still] say that there is at present no occasion and no reason to speak of causality in nature – because no experiment indicates its presence, since the macroscopic are unsuitable in principle, and ... quantum mechanics contradicts it ([6], pp. 326–327).

As we have seen, the cost of these sweeping assertions is to force the observer (i.e. the subjective) to intrude into the objective world, and indeed, in a way which makes the intrusion itself objective. We are thus mandated to do precisely the thing which 'objectivity' denies; namely, to involve the subjective observer in accounting for what is observed. That is, the boundary between subjective and objective has become porous. And it is equally clear that the more we try to include on the 'objective' side of our boundary (i.e. the more microscopic a view we take, and the more of 'the body of the observer' we include in it) the more porous it gets; the less of a separation it actually makes.

The real problem lies in trying to tie 'objectivity' irevocably to a putative notion of state, and thereby restricting 'causality' to pertain *only* to (a) the determination of observables, and (b) the determination of state transitions, by state. In fact, causality, in the original Aristotelian sense, means much more than this. Accordingly, we shall argue that any science which mandates this, including quantum theory, must be a very special science; in particular, it will have its own version of 'the measurement problem' or of the life-organism problem, or the mind-brain problem.

Of course, in the above few pages we can hardly do justice to the ramifications of 'the measurement problem'. For a survey see e.g. [7].

COGNATE ANOMALIES IN MATHEMATICS

The domain of Mathematics lies entirely within the inner, private, subjective world; ironically, however, that domain is also considered the most 'objective' of realms. From at least the time of Pythagoras, 'mathematical truth' was the best truth, independent not only of the mathematician but of the external world itself. A culmination of this was the development of Platonic Idealism, in which a 'real thing' is regarded as something mathematical (its Idea) plus the corruption occasioned by attaching that Idea to a specific (hence 'imperfect') external referent. Materialism, in any of its many forms, can likewise be regarded as an opposite attempt to pull this 'mathematical truth' into conventional perceptive realms arising in the external world. We can see in this opposition the germs of the mind-brain problem itself.

In any event, the mathematical universe comprises systems of entailment (inferential entailment) no less compelling than the causal relations governing objective events in the external world. Indeed, inferential entailment (between propositions) and causal entailment (between external events) are the only two modes of entailment we know about. We can in fact deal with both of them in exactly parallel Aristotelian terms, by asking "Why?" s.

The surprising fact is that these two different realms of entailment (the objective world of causal entailment, and the relatively subjective realm of inferential entailment) run so much in parallel. The physicist Wigner [8] thought of this as *unreasonable*, and it is still a matter of lively debate (cf. e.g. [9]). In fact, the congruences (modelling relations) which can be established between them are, I would argue, the essential stuff of science (cf. [4]).

Over the past century, the mathematical realm has run into a great deal of trouble; a Foundations Crisis, just as physics has. This current Crisis was touched off by the appearance of paradoxes, some of which (e.g. the 'liar paradox' of Epimenides) actually go back to ancient times.

Most, if not all, the known paradoxes arise from an attempt to divide a universe into two parts on the basis of satisfying some property or not (e.g., a property like 'objectivity'). *Trouble arises whenever this property can be turned back on itself*; in particular, when we try to put some consequent of the property back into one or the other class defined *by* the property. This constitutes an *impredicativity*; what Bertrand Russell called a *vicious circle*. Kleene puts the matter as follows:

When a set M and a particular object m are so defined that on the one hand m is a member of M, and on the other hand the definition of m depends on M, we say that the procedure (or the definition of m, or the definition of M) is *impredicative*. Similarly, when a property P is possessed by an object m whose definition depends on P (here M is the set of the objects which possess the property P). An impredicative definition is circular, at least on its face, as what is defined participates in its own definition ... In the Epimenides paradox [i.e. Epimenides the Cretan says that all Cretans are liars] the totality of statements is divided into two parts, the true and the false statement. A statement which refers to this division is reckoned as of the original totality, when we ask if it is true or false ([10], p. 42).

On the other hand, as Kleene states,

Thus it might appear that we have a sufficient solution and adequate insight into the paradoxes *except for one circumstance*; parts of mathematics we want to retain, particularly analysis, also contain impredicative definitions. An example is the definition (of the least upper bound of an arbitrary bounded set of real numbers) ... ([10, p. 42; emphasis added).

What has come to be called *constructible mathematics* is an attempt to eliminate all the inferential loops or 'vicious circles', thereby on the one hand eliminating the basis of all the paradoxes, and on the other, providing equivalent *predicative* definitions of presently impredicative ones, like 'least upper bound', (which, it may be noted, is the basis for every approach to *optimality*, among many other things). It was widely supposed that *only* the things in constructive mathematics *had any 'objective' basis for existing*.

Russell's 'Theory of Types' represents one kind of attempt to straighten out all the impredicative loops of inferential entailment in Mathematics. It was a failure, which lost itself in at least equally bad infinite regresses of unlimited complication. Another, more modern attempt in this connection, relies on a 'constructible universe', originally proposed by Gödel as a *model* for set theory, in connection with determining the status of such things as the Axiom of Choice and the Continuum Hypothesis.

Such a 'constructible universe' is one which starts with a finite (usually small) number of elementary syntactic operations, and a minimal class of generators for them to operate on. Time moves in discreet ordinal steps. At each stage, new things can be constructed, by applying one of the rules to what has been constructed in the preceding stages. Thus, no closed loops, no impredicativities can arise in that universe. In particular, anything in that universe has a pedigree, going back to the original elements, through specific rules applied at successive ordinal time-steps; i.e. it is *algorithmically* generated.

Without going into details, we can see already the machine-like character of this kind of 'constructible universe'. This is precisely what is supposed to make it 'objective'. As in the causal world, no later stage can affect *any* earlier one; certainly not the original generators of which it is comprised.

Another language to describe this kind of 'constructible universe' is that of *formalization.* A mathematical system is formalizable if everything in it can be generated in such a constructible fashion. Formalization was Hilbert's answer to the paradoxes, and consisted in essence of stripping the system of all referents (even mathematical ones). In effect, mathematics was to become a game of pattern generation, played by applying syntactic rules governing the manipulation of the *symbols* in which the mathematics was expressed. To quote Kleene again:

> [Formalization] will not be finished until all the properties of undefined terms which matter for the deduction of theorems have been expressed by axioms. Then it should be possible to perform the deductions treating the technical terms as words in themselves without meaning. For to say that they have meanings necessary to the deduction of the theorems, other than what they derive from the axioms which govern them, amounts to saying that not all of their properties which matter for the deductions have been expressed by axioms. When the meanings of the technical terms are thus left out of account, we have arrived at the standpoint of formal axiomatics ([10], p. 59).

Where there are no referents at all, there are *a fortiori* no self-referents. Moreover, the internalizing of such referents in the form of additional syntactic rules is precisely the basis for regarding formalizations as 'objective'. Once again, no impredicativities can arise in such a formalization. It was Hilbert's program to thus formalize all of mathematics.

The status of all these formalizations is informative. They turn out to be infinitely feeble compared with the original mathematical systems they attempted to objectivize. Indeed, these attempts to secure Mathematics from paradox by invoking constructability, or formalizability, end up by losing most of it. This is one of the upshots of Gödel's celebrated Incompleteness Theorem [11], which showed precisely that 'self-referential' statements (e.g. "this proposition is unprovable in a given formalization"), which are perfectly acceptable in the context of ordinary Number Theory, fall outside that formalization. In other words, a 'constructible universe' is at best only an infinitesimal fragment of 'mathematical reality', considered as a system of inferential entailment, no matter how many elements, or how many syntactic rules, we allow into it.

This situation should remind the reader of what we have already seen. We can push the boundary between what is constructive, or formalizable, arbitrarily far into Mathematics. But wherever we draw that boundary, more remains on the wrong side of it. On the formalizable ('objective') side of the boundary, there are no closed loops of entailment, just as, in the causal realm, we exclude them by restricting ourselves to those causal categories (material, formal and efficient causes) which only go in a fixed time-frame from earlier to later. Thus we can see, in particular, that there is no way to ever map the nonformalized side into the formalized one *in an entailment-preserving fashion*. There are many ways to say this: we cannot replace semantics by syntax; Mathematics is much more than word processing or symbol manipulation; Mathematics transcends algorithms; Mathematics cannot be expressed as program to fixed, finite-state hardware.

In a certain sense, then, formalizable mathematical systems (i.e. those without impredicativities) are indeed infinitely feeble in terms of entailment. As such, they are excessively non-generic; infinitely atypical of mathematical (inferential) systems at large, let alone 'informal' things like natural languages. Any attempt to objectify all of mathematics by imposing some kind of 'axiom of constructibility', or by invoking Church's Thesis (cf. [12, 13]), only serves to estrange one from Mathematics itself. Indeed, what is necessary is quite the opposite; *it is to objectify impredicative loops* (cf. e.g. [14]).

IMPLICATIONS FOR THE MIND-BRAIN PROBLEM

We will now try to pull together the various threads we have developed above, and indicate their bearing on the mind-brain problem.

From what has been said above, the 'objectivizing' of the observer (i.e. pulling him entirely into the public, external world) amounts to replacing the boundary between subjective and objective by an ordinary boundary between a 'system' and its environment, both now in the external world. Moreover, this must be done in such a way that what, formerly, was (subjective) inferential entailment in the observer now *coincides* with causal entailment in the 'objective' system which has replaced him. At the very least, there must be no less *causal* entailment in that system than there was *inferential* entailment in the subjective observer.

These requirements are inconsistent with the tenets of mechanism; tenets which, as we have seen, have been presumed synonymous with 'objectivity' itself.

For instance, 'mind' requires unformalizability to be part of it. That means, precisely, the accommodation of closed loops of inferential entailment. But as we have seen, the identification of mechanism with 'objectivity' forbids closed *causal* loops. And yet the presumed 'objectivization' of the observer must faithfully represent his inferential entailments in terms of objective causal ones.

That is one thing, which by itself would be quite decisive. But there are many others. For instance: part of the subjective world must comprise the *referents* he establishes between his internal models and the systems he sees in the external world. It is clear that any attempt to 'objectivize' these creates an immediate *self*-reference *in the presumed objectivization*, where no referents are allowed at all. Indeed, this situation spawns actual paradoxes, involving mappings forced to belong to their own domains and ranges (cf. [15]).

The conclusion I draw from these circumstances is that, so long as we persist in equating mechanism with 'objectivity', and hence with science, the mind-brain problem (and even more, the life-organism problem) are inherently outside the reach of that science. The alternatives, in the starkest possible form, are either to give up the problem, or to change the science. As to the latter, the change required is clear, it is abandoning the equation of 'objectivity' with mechanism. We must accordingly allow an 'objective' status to complexity as defined above; i.e. to systems which can accommodate impredicativies, or closed loops of entailment.

Actually, such an enlargement seems much more radical in concept than it would be in practice. Mechanism would not thereby disappear; it becomes simply a limiting case of complexity. So, for example, the discovery of irrational numbers was a much more radical conceptual change than a practical one. Nevertheless, moving from rational numbers to real numbers was a revolutionary step, certainly in terms of the conceptual possibilities it opened up. So it is here.

Let me address one objection, which is always raised at this point. It is this. Surely, each individual particle in 'the body of the observer' is a mechanical thing. And (at least at any instant) there are only a finite number of them. Hence, if we pull all these constituent particles into the 'objective world', one by one, the observer himself will necessarily come with them. Hence the observer is himself mechanical, and indeed could be objectively re-created, particle by particle. In particular, we could re-create a subjective mind by thus assembling an objective brain.

There are many flaws in such an argument. The most cogent is that the putative assembly process, the process of re-creation itself, has become complex; hence non-"objective". In addition, such a process would actually obliterate the mind-brain interface (or, more generally, the subjective-objective boundary), which is now merely a system-environment boundary. Obliterating the boundary would leave us with the entire universe; either all environment and no system, or all system and no environment. And we recall that, as von Neumann argued, such a boundary must be put somewhere, "if the method is not to proceed vacuously" [6].

It should be stressed that, by advocating the 'objectivity' of complex systems, systems with non-formalizable models and hence closed loops of entailment (impredicativities), I am advocating the objectivity of at least a limited kind of *final* causation. This is precisely what closes the causal loops. It simply describes something in terms of what it entails, rather than exclusively in terms of what entails it. This, it will be observed, need have nothing whatever to do with *Telos*, any more than, say, Gödel's Incompleteness Theorem does.

In such a complex world, furthermore, functional descriptions are perfectly meaningful, and can be quite independent of any mechanistic ones. And, since we are freed from the exigencies of a single constructive or algorithmic time-frame, mechanistic objections to anticipation, and in particular to internal predictive self-models which provide its basis, no longer apply at all. I have dealt with all these matters at greater length elsewhere (cf. [5]); and shall do so again in another place.

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