Algorithms in Genome Research
Winter 2024/2025

Exercises

Number 3, Discussion: 2024 November 15

1. Given the following three permutations of the numbers in the set $\{1,\ldots,9\}$:

$$A = [1,2,3,4,5,6,7,8,9]$$
$$B = [9,8,4,5,6,7,1,2,3]$$
$$C = [1,2,3,8,7,4,5,6,9]$$

   Find all their common intervals. Which of them are irreducible, which are reducible?

2. Given two linear genomes (represented as sequences of gene families):

$$A = [1,4,7,6,5,4,5,1,4,3,2]$$
$$B = [1,2,3,4,3,4,5,7]$$

   (a) Find all common intervals of A and B of size at least 2 and list all their occurrences in A and in B. Which of these occurrences are maximal?

   (b) Find five maximal median gene clusters of A and B of size at least 3 with a symmetric set distance of at most 1.

3. A common interval C is called a *nested common interval* of two genomes if either $|C| = 2$, or if $|C| > 2$ and it contains a nested common interval of size $|C| - 1$.

   A nested common interval of size ℓ is *maximal* if it is not contained in a nested common interval of size $\ell + 1$.

   (a) Find all maximal nested common intervals in the following two linear genomes:

$$A = [4,6,5,7,3,1,2]$$
$$B = [1,2,3,4,5,6,7]$$

   (b) Develop an algorithm to find all maximal nested common intervals in two permutations.

4. Given two linear genomes (represented as indeterminate strings):

$$A = [\{9\},\{3,7,10\},\{8,13\},\{4,6\},\{11\},\{1,9\},\{2\}]$$
$$B = [\{5\},\{7\},\{10\},\{4,8,12\},\{6\},\{3\},\{2,10\},\{1\},\{12\}]$$

   Find all weak common intervals of A and B of size at least 2.

5. Obviously there exist bijective mappings between the numbers $1,2,\ldots,n!$ and the (unsigned) permutations over $\{1,2,\ldots,n\}$. Find such a mapping that is computable in both directions in polynomial time.