Algorithms in Comparative Genomics

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Dr. Marília D. V. Braga · Prof. Dr. Jens Stoye

https://gi.cebitec.uni-bielefeld.de/teaching/2025summer/cg

Exercise sheet 11, discussion: 11.07.2025

Exercise 1 (Common Intervals of Two Permutations)
Given the following two permutations of the numbers in the set $\{1, \ldots, 12\}$:

$$\begin{aligned} \mathbb{A} &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] \\ \mathbb{B} &= [8, 4, 9, 5, 7, 6, 2, 10, 11, 1, 12, 3] \end{aligned}$$

Execute the algorithm discussed in class that runs in $O(n^2)$ time where n is the length of the permutation. How many common intervals do you find?

Exercise 2 (Common Intervals of Three Permutations)
Given the following three permutations of the numbers in the set $\{1, \ldots, 9\}$:

$$\begin{aligned} \mathbb{A} &= [1, 2, 3, 4, 5, 6, 7, 8, 9] \\ \mathbb{B} &= [9, 8, 4, 5, 6, 7, 1, 2, 3] \\ \mathbb{C} &= [1, 2, 3, 8, 7, 4, 5, 6, 9] \end{aligned}$$

Find all their common intervals. Which of them are irreducible, which are reducible?

Exercise 3 (Common Intervals of Two Sequences)
Given two genomes (represented as gene sequences):

$$\begin{aligned} \mathbb{A} &= [3, 4, 5, 6, 5, 4, 7, 4, 1, 2] \\ \mathbb{B} &= [1, 2, 2, 3, 4, 3, 4, 5, 7] \end{aligned}$$

Find all maximal common intervals of $\mathbb{A}$ and $\mathbb{B}$ of size at least 2.

Exercise 4 (Nested Common Intervals)
A common interval C is called a *nested common interval* of two genomes $\mathbb{A}$ and $\mathbb{B}$ if either $|C| = 2$, or if $|C| > 2$ and it contains a nested common interval of size $|C| - 1$.
A nested common interval of size ℓ is *maximal* if it is not contained in a nested common interval of size $\ell + 1$.

1. Find all maximal nested common intervals in the following two genomes:

$$\begin{aligned} \mathbb{A} &= [4, 6, 5, 7, 3, 1, 2] \\ \mathbb{B} &= [1, 2, 3, 4, 5, 6, 7] \end{aligned}$$

2. Develop an algorithm to find all maximal nested common intervals in two permutations.