Algorithms in Genome Research
Winter 2025/2026

Exercises

Number 10, Discussion: 2026-January-23

1. Using only basic data structures (lists, arrays), devise a linear time algorithm for computing the breakpoint distance $d_{BP}(\mathbb{A}, \mathbb{B})$, where $\mathbb{A}$ and $\mathbb{B}$ form a pair of canonical genomes that can contain multiple linear or circular chromosomes.

2. Write down all string representations for the following singular chromosomes.

   - $C_1 = [1\,4\,\bar{5}\,\bar{2}\,3]$
   - $C_2 = (1\,\bar{3}\,2)$

   Develop a general formula for the number of string representations of singular chromosomes.

3. Consider the following two canonical genomes:

$$\mathbb{A} = \{\ [3\ 4\ \bar{8}]\ \ [2\ 1\ 7\ 5\ 6]\ \ (11\ 12\ \overline{13}\ 9\ 10)\ \ [\overline{15}\ \overline{14}]\ \}\ \text{and}$$
$$\mathbb{B} = \{\ [1\ 2\ 3\ 4\ 5]\ \ (6\ 7)\ \ [8\ 9\ 10\ 11\ 12\ 13\ 14\ 15]\ \}.$$

   (a) Draw the relational graph of $\mathbb{A}$ and $\mathbb{B}$.
   (b) What is the breakpoint distance between $\mathbb{A}$ and $\mathbb{B}$?
   (c) What is the SCJ distance between $\mathbb{A}$ and $\mathbb{B}$?
   (d) Explain the difference between the two distances (if any).

4. A theoretical lower bound for the SCJ distance with respect to the breakpoint distance is

$$d_{BP}(\mathbb{A}, \mathbb{B}) \leq d_{SCJ}(\mathbb{A}, \mathbb{B}).$$

   (a) Give an example of a pair of mutually distinct genomes showing that this bound is tight.
   (b) Determine a tight upper bound for the SCJ distance with respect to the breakpoint distance and explain your answer.

5. Consider the following two balanced genomes:

$$\mathbb{A} = \{\ [1\ 3\ 1\ 2\ 4\ 6\ 6\ 2\ \bar{6}\ 4\ 5]\ \}\ \text{and}$$
$$\mathbb{B} = \{\ (2\ 1\ \bar{5}\ 6)\ [2\ 4\ 6\ 1\ \bar{4}\ \bar{3}\ 6]\ \}.$$

   (a) How could the distance of balanced genomes be phrased in terms of the distance of canonical genomes?
   (b) Give the number of distinct pairs of canonical genomes that can be obtained by fixing a maximal matching of the genes of $\mathbb{A}$ and $\mathbb{B}$.